MATHEMATICAL MODELLING FOR RUDDER ROLL STABILIZATION

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ABSTRACT

Modern passenger ships as well as naval ships are equipped with roll stabilization systems in order to improve the passenger's comfort or to keep the ship fully operational in bad weather conditions. Fins and tanks are most commonly used but both have disadvantages. Tanks require a lot of space, fins introduce a considerable drag and are expensive. Besides, fin motions disturb the heading control system, while rudder motions not only effect a ship's heading but influence the rolling motions as well. In present systems this interaction is generally disregarded. However, by explicitly modelling the interaction it can purposefully be used by applying the rudder for roll stabilization as well. This paper describes a simple mathematical model for the transfer between the rudder angle and the two outputs: rate of turn and roll angle. The parameters of this model can be estimated from full-scale trials such as zig-zag manoeuvres. Examples are given of the parameter estimation of two different ships, a pilot vessel and a naval ship.

INTRODUCTION

Since a long time ago automatic control systems have been applied to controlling the motions of a ship. In most cases an autopilot for controlling the heading has replaced the helmsman, although manual steering remains possible. To reduce the rolling motions, tanks and fins have been applied which always work fully automatically. Until recently the controller structure of these systems was simple. But the availability of small and inexpensive digital computer systems offers a possibility to apply more advanced control algorithms into a wide range of practical systems. This has already led to a series of new autopilot designs, which all claim more accurate and more economical control of a ship's heading, by introducing adaptive properties into the controller (see for instance Van Amerongen and Van Nauta Lemke, 1978; Van Amerongen , 1981).

Although an autopilot which generates only smooth rudder motions implicitly causes less roll, this is seldom explicitly used as a design criterion. On the other hand, the coupling between the stabilizer fins and yawing are disregarded in the design as well. To get an optimal performance of both systems the ship should be considered as one multi-variable system with two inputs: rudder angle and fin angle, and two outputs: heading and roll angle; one integrated controller should be designed for both actuators.

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Another possibility with promising properties is to use the rudder not only for control of the heading but for roll stabilization as well (Carley, 1975; Cowley and Lambert, 1972, 1975; Lloyd, 1975, Baitis, 1980). Although this will require a more powerful steering machine the savings realized by not installing stabilizer fins are apparent. Also with respect to fuel economy a rudder roll stabilization (RRS) system may be advantageous. This aspect is of growing importance. For merchant ships the total operational cost is already for more than sixty percent determined by the fuel cost. (See figure 1 , according to Milch, 1980.)

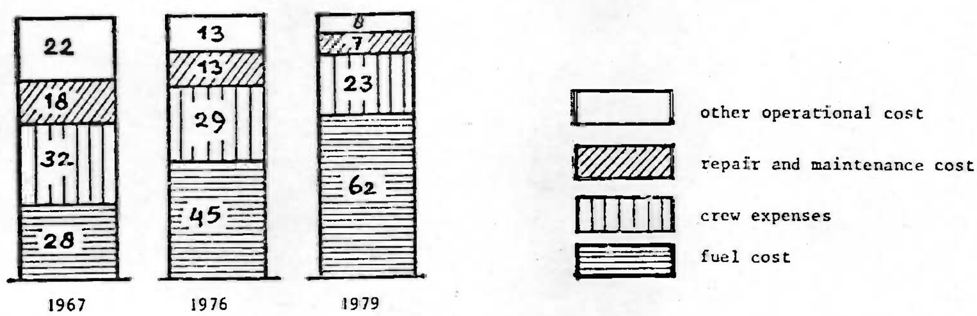

Figure 1 Increasing importance of fuel cost

Rough estimates indicate that the loss of speed due to the drag of the stabilizer fins is approximately ten percent. Recently several papers have discussed a perfomance criterion for a course autopilot (Koyama, 1967; Norrbin, 1972; Van Amerongen and Van Nauta Lemke, 1980). It can be shown that the loss of speed is minimized by minimizing the rate of turn of a ship, for instance by applying only small and smooth rudder motions. However, the rudder itself causes only a neglectable small drag. From the data provided by Norrbin, 1972 it follows that for a cargo liner with 33000 tons displacement and a length of 200 meters the loss of speed due to steering is described by (Van Amerongen and Van Nauta Lemke, 1980) :

$$\frac{0.0076}{T}\int_0^T (\varepsilon^2 + 1600\dot{\psi}^2 + 6\delta^2)\,dt \quad \% \qquad (1)$$

The loss of speed caused by the rudder only is thus:

$$\frac{0.0076}{T}\int_0^T (6\delta^2)\,dt \quad \% \qquad (2)$$

A rudder angle of, for instance, 10 degrees gives a loss of speed of nearly five percent, supposed that the ship does not start turning.

It can be shown that for control of the heading high-frequency rudder motions have no positive effect on the course-keeping accuracy (Van Amerongen and Van Nauta Lemke, 1980): course control only necessitates low-frequency rudder motions. With respect to the frequencies of these motions the rolling motion is high frequent. Quick rudder motions, to suppress the rolling motion, with a mean value computed by the course controller, will therefore hardly influence the ship's heading. Because of eqn.'s (1) and (2) the loss of speed caused by these quick rudder motions can be kept on a reasonable value as long as turning is prevented.

MATHEMATICAL MODELLING

The basic equations which describe the motions of a ship, important with respect to steering and roll stabilization are:

$$Y = m\,(\dot{v} - ur) \qquad (3)$$

$$K = I_x \ddot{\varphi} \qquad (4)$$

$$N = I_z \dot{r} \qquad (5)$$

where m is the ship's mass, included the added mass of the water.

Ix and Iz are the moments of inertia about the x-axis and z-axis.

Y is the hydrodynamic force in the y-direction.

K and N are hydrodynamic moments.

The other variables have been defined in figure 2a and 2b.

The eqn.'s (3) - (5) can be expanded into a Taylor series. See for instance Eda, 1978. Disregarding all higher order terms and introducing the fin angle α yields the following simplified equations:

$$Y = Y_v v + Y_r r + Y_\varphi \varphi + Y_\delta \delta + Y_\alpha \alpha \qquad (6)$$

$$K = K_v v + K_r r + K_\varphi \varphi + K_{\dot{\varphi}} \dot{\varphi} + K_\delta \delta + K_\alpha \alpha \qquad (7)$$

$$N = N_v v + N_r r + N_\varphi \varphi + N_\delta \delta + N_\alpha \alpha \qquad (8)$$

Substitution of eqn.'s (3) and (6) into (7) and (8), and substitution of eqn. (4) into (7) and eqn (5) into (8) yields, after Laplace transformation:

$$\varphi = \omega_n^2 \frac{K_\delta \delta + K_\alpha \alpha - K_r r}{s^2 + 2z\omega_n s + \omega_n^2} \qquad (9)$$

$$r = \dot{\psi} = \frac{n_\delta \delta + n_\alpha \alpha - n_\varphi \varphi}{\tau_r s + 1} \qquad (10)$$

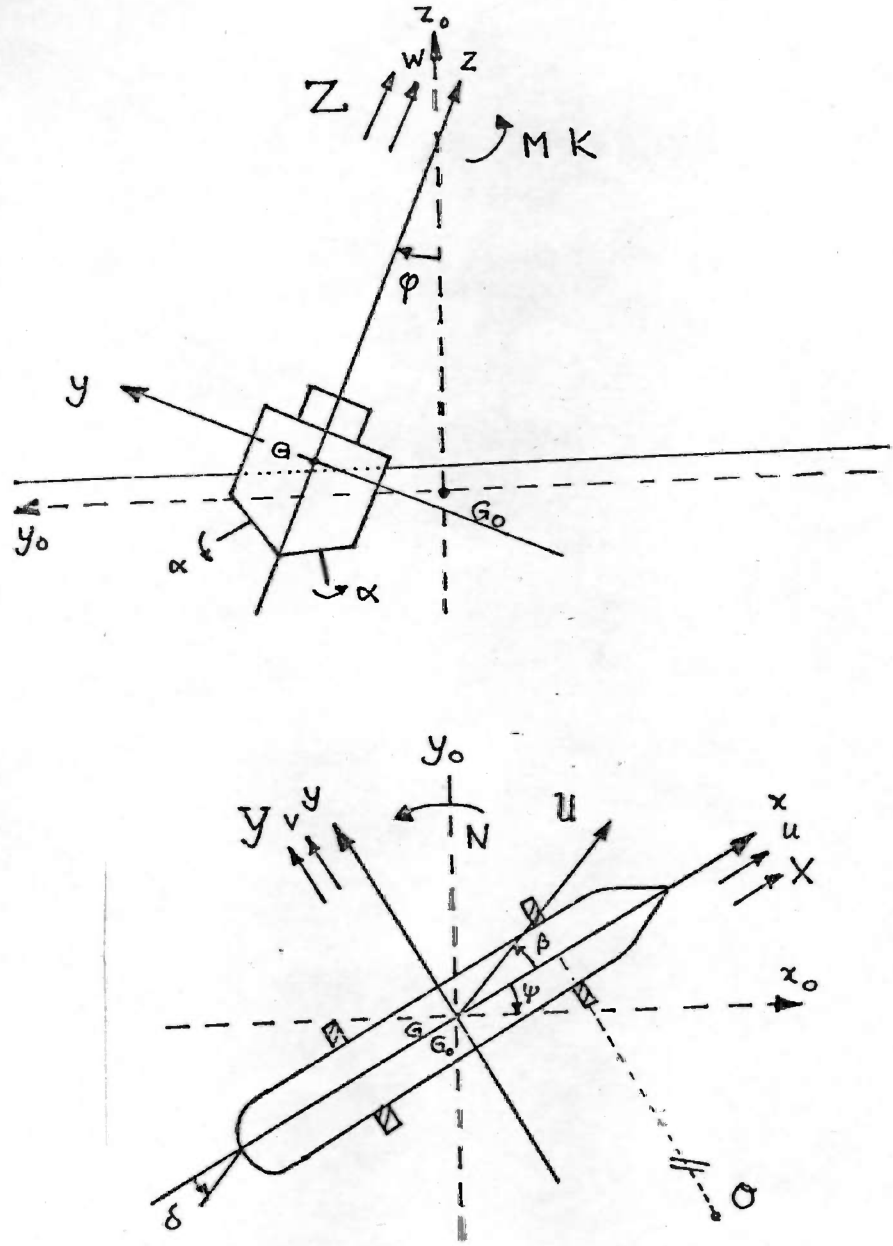

Figure 2 Definition of variables

Disregarding the influence of the fins and the roll angle on the rate of turn, eqn. (10) transforms into the well known Nomoto model. Equations (9) and (10) can be combined into one block diagram as shown in figure 3.

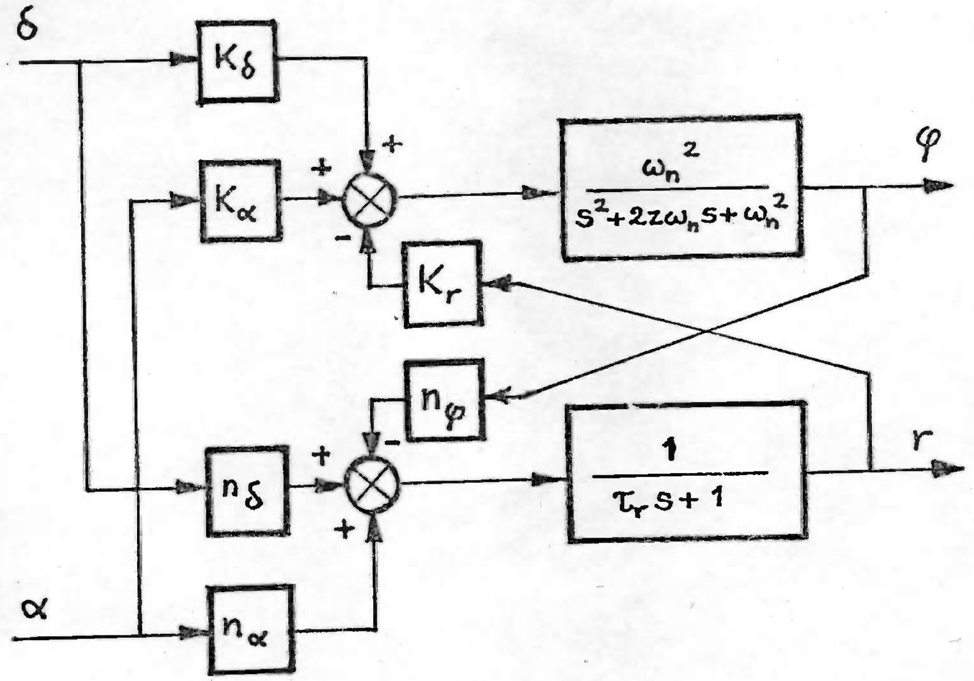

Figure 3 Blockdiagram of the dynamics between rudder and roll

When the ship has no stabilizing fins and the coupling of φ and r is disregarded, this blockdiagram simplifies into the system of figure 4. In figure 4 the parameters K_N and τ_N have raplaced n_δ and τ_r. This model will be useful for investigation of rudder roll stabilization systems.

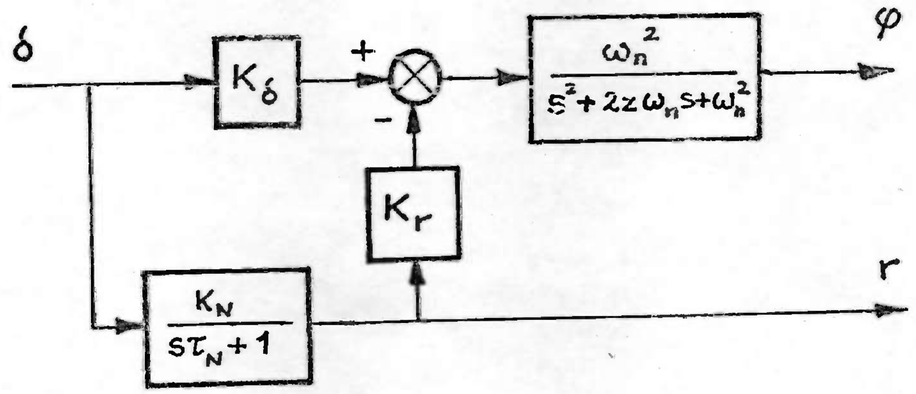

Figure 4 Simplified blockdiagram

PARAMETER ESTIMATION FROM FULL-SCALE TRIALS

The simplicity of the model of figure 4 enables to estimate its parameters from full-scale trials. Zig-zag manoeuvres are well suited as test signals. During the trials the rudder angle, the rate of turn and the roll angle should be recorded. This enables a two-stage identification procedure:

1. Determine the parameters K_N and τ_N from the rate of turn and rudder signals.

2. Use the rate of turn signal computed by the now identified Nomoto model, together with the rudder and roll angle signals to estimate K_δ, K_r, z and ω_n.

For both stages hillclimbing with the aid of a digital computer works well. In case that the circumstances are not ideal, for instance when there is wind, it is necessary to estimate two additional constants r_0 and φ_0, which must be subtracted from the measured r and φ signals. For obtaining accurate results the constants r_0 and φ_0 should be small.

The parameter-estimation procedure was tested on data which were available from earlier measurements with a pilot ship. It appeared that for this ship the second order part of the transfer function could be well approximated by one single pole. This yields the block diagram of figure 5.

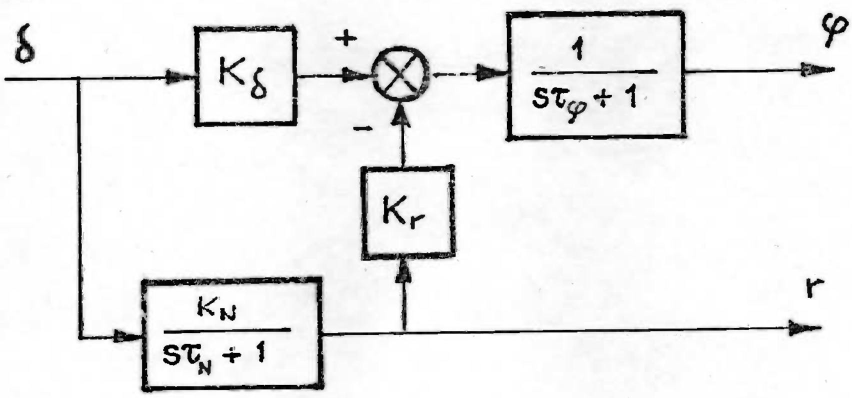

Figure 5 First order rol dynamics

For this pilot ship, with a lentgh of 60 meters and sailing with a speed of 12 knots the parameters are given in table 1.

The same procedure was used to estimate the parameters of a naval ship, about twice as long and sailing with a speed of 21 knots. For this ship the parameters of the model of figure 5 have also been determined, but the responses clearly indicated the need of using the second-order roll dynamics of the model of figure 4. Parameters of both models are given in table 2.

In figures 6 and 7 the measured responses and model responses are given for the pilot ship, (first-order roll dynamics) and for the naval ship (second-order roll dynamics).

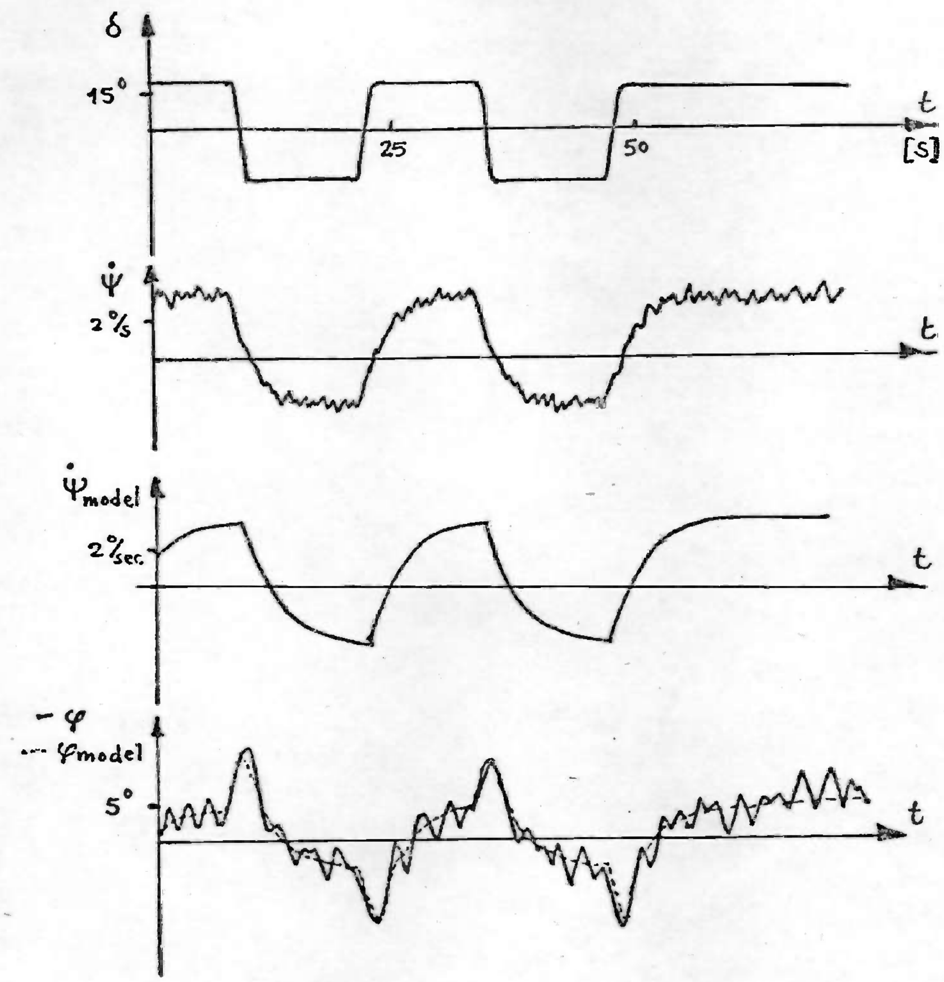

Figure 6 Results of identification of a pilot vessel

Table 1 parameters of a pilot vessel

yaw dynamics	roll dynamics
$K_N = 0.125$	$K_\delta = 0.4$
$\tau_N = 10$	$K_r = 6$
	$\tau_\varphi = 1.9$

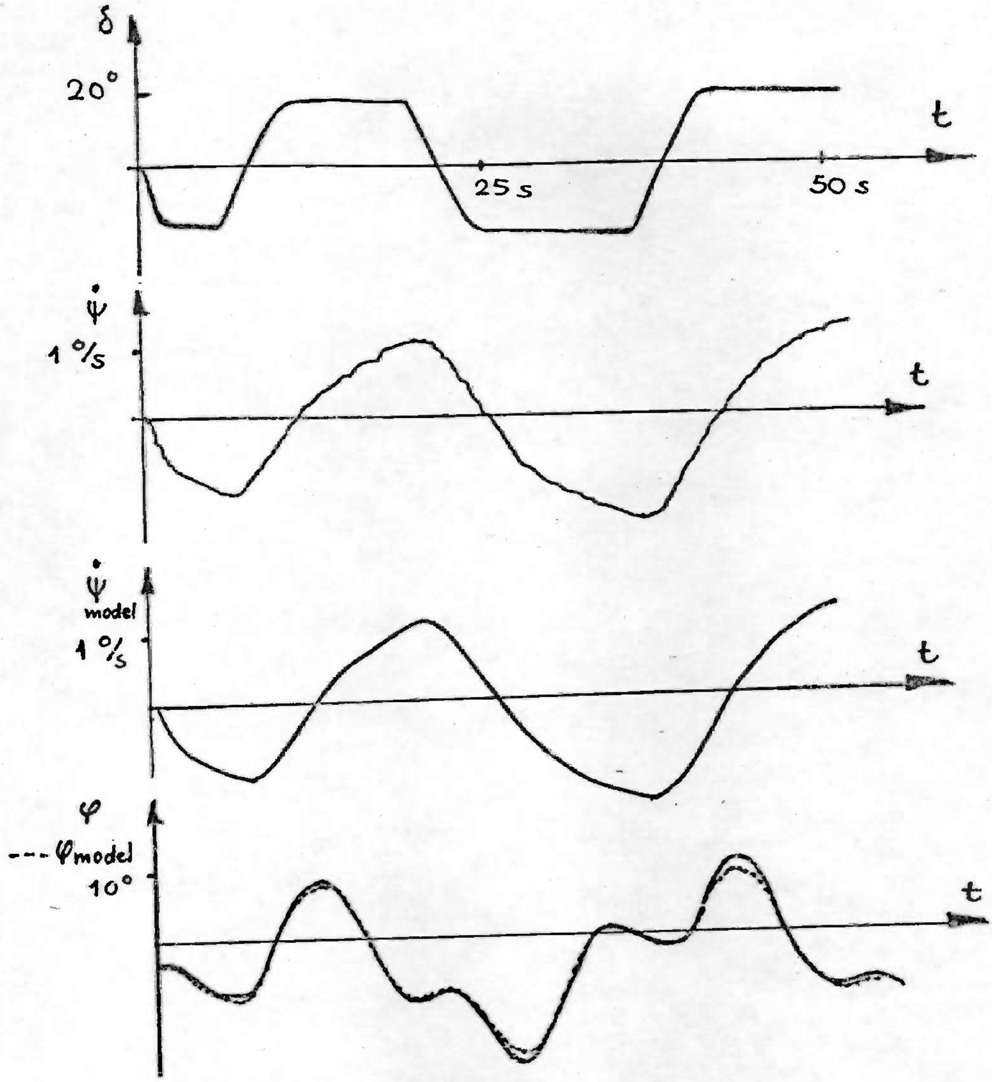

Figure 7 Results of identification of a naval ship

Table 2 parameters of a naval ship

yaw dynamics	first-order roll dynamics	second-order roll dynamics
$K_N = 0.09$	$K_\delta = 0.22$	$K_\delta = 0.24$
$\tau_N = 6$	$K_r = 5.3$	$K_r = 5.4$
	$\tau_\varphi = 1.9$	$z = 0.23$
		$\omega_n = 0{:}55$

CONCLUSIONS

It has been shown that relatively simple models can be derived to describe the transfer between rudder and roll. The parameters of these models can be estimated from full-scale zig-zag manoeuvres. For some ships a model with first-order roll dynamics appears to give a reasonably good description.

The models also give some insight into the ability of the rudder to stabilizing a ship's roll. Due to the non-minimum phase character of the responses the rudder will never be able to compensate a stationary roll angle, what fins are able to do. Only in the high frequency range the rudder has the desired effect. For low frequencies the roll in opposite direction, caused by the rate of turn will be dominant. However, als the course control system requires the rate of turn to be kept small.

Figure 8 shows bode diagrams for the transfers between rudder and heading and between rudder and roll, calculated with the second-order roll parameters of table 2. The low-frequency character of the rudder-heading transfer function and the more high-frequency character of the rudder-roll transfer function can clearly be seen.

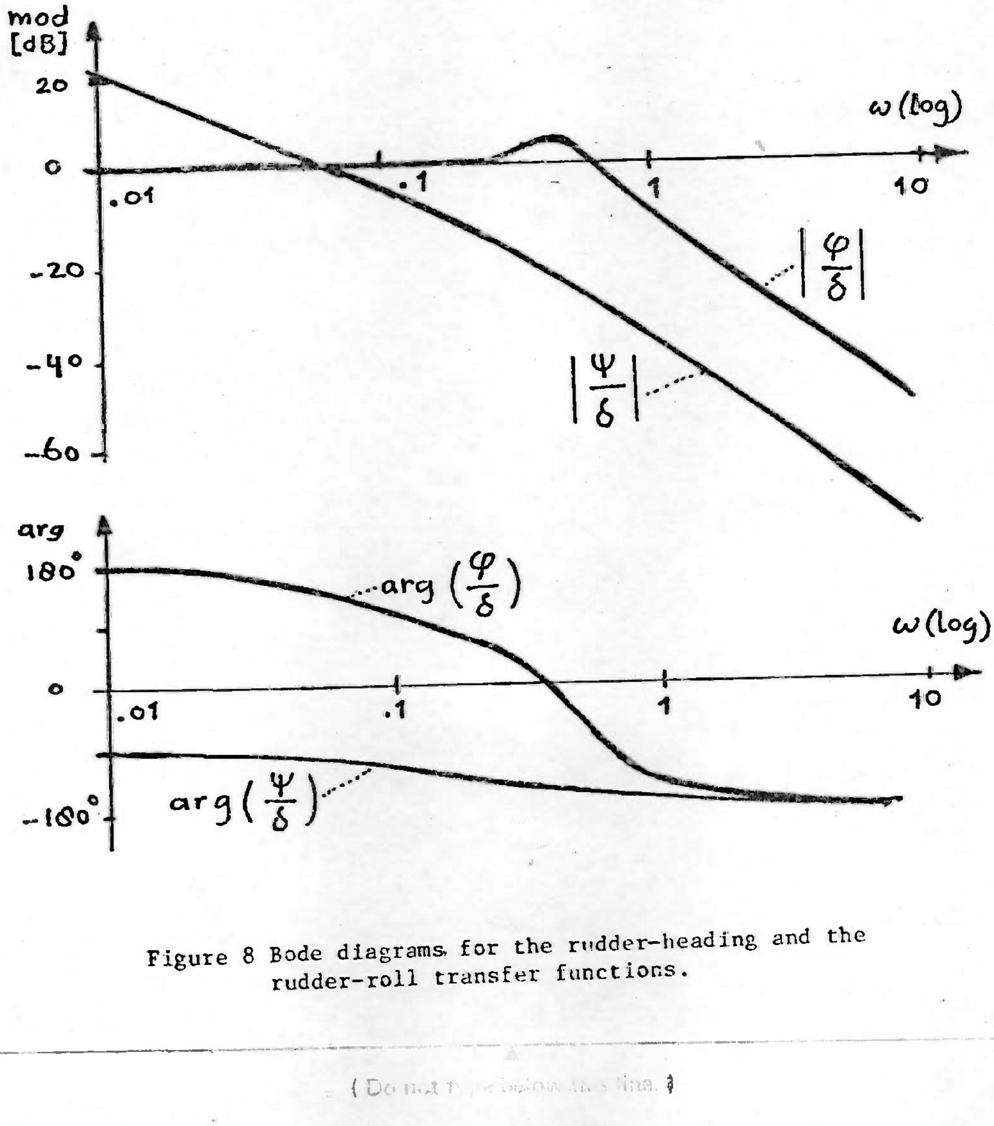

Figure 8 Bode diagrams for the rudder-heading and the rudder-roll transfer functions.

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